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TECHNICAL MEMORANDUM 774

NOVEMBER 1967

# JAMMER TRACKING WITH FREQUENCY-SCANNED RADARS BY LOBE COMPARISON

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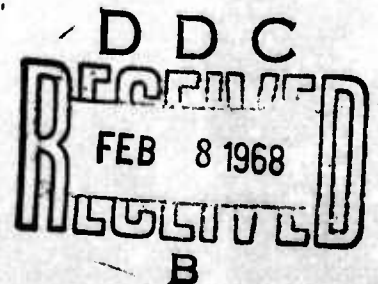
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## ABSTRACT

Frequency-scanned array radars may be preferred to phase-scanned radars because they tend to be cheaper and simpler. When conventional frequency-scanned radars are used to measure the angular position of jammers, however, large errors can result. These can be eliminated with the lobe-comparison technique.

The response of a frequency-scanned array to a jammer can be compared with that of a filter whose characteristics depend on the angle of signal arrival. Variations of jammer power with frequency and time can distort the array output spectrum to cause measurement errors. With lobe comparison, the array is divided to produce two slightly different frequency response functions (equivalent to two beams, like those in a monopulse radar). The angle of signal arrival is estimated by comparing the response of the two arrays in a narrow frequency band. If the band is sufficiently narrow, measurement error can be reduced to a satisfactory level with no penalty in noise performance. In practice, a bandwidth about one-fifth that of the radar signal reduces error to less than the usual fixed-error limit of the array.

Lobe comparison can be applied to frequency-scanned radars, including phase-frequency-scanned arrays, with only a small increase in cost and complexity. The improved performance in measuring jammer angular position compares well with that of phase-scanned arrays.

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## I. INTRODUCTION

Frequency-scanned array radars make use of fixed-time delays between adjacent array elements to produce an interelement phase difference that depends on signal wavelength; the beam pointing angle is then controlled by the frequency of signal transmission or reception. Usually cheaper and simpler, these radars are often preferred to comparable phase-scanned arrays, which use electronically variable phase shifters to control the phase of each array element.

Nevertheless, conventional frequency-scanned arrays may be unsuited to some military applications, because the interrelation between frequency and scan angle can cause large errors in measuring the angular position of noise jammers. The causes of these errors, unique to frequency-scanned radars, are discussed in Sec. II, together with a technique for their elimination.\*

In a conventional frequency-scanned radar, the angular position of a jammer can be determined only by observing jammer power in several frequency bands;\*\* variations in jamming power with frequency and time cause errors in the measurements. With lobe comparison,\*\*\* simultaneous observations are made in a single frequency band; this technique eliminates errors caused by power variations with either time or frequency and leaves angle-measurement errors no larger than the residual error from array component tolerances. Furthermore, this technique can be incorporated into frequency-scanned radars with only a small increase in cost and complexity.

Radars that scan in two angular coordinates often employ frequency-scanning in one coordinate and scanning controlled by electronically

---

\* Conventional frequency-scanned radars may also produce large target-angle-measurement errors for non-jamming targets; these errors, too, are eliminated by this technique.

\*\* In some radars these observations are made at different times.

\*\*\* Lobe comparison is analogous to monopulse measurement, except that the signal is not pulsed.



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variable phase shifters in the other; these are sometimes called phase-frequency-scanned arrays. When the lobe comparison technique is applied to these radars,<sup>\*</sup> they can not only measure the angle of signal arrival as accurately as can a planar phased array, but they usually cost less and are simpler.

---

<sup>\*</sup> An example is given in Sec. III.

## II. DISCUSSION

### A. ARRAY FREQUENCY RESPONSE

A frequency-scanned array can be considered as a frequency-selective filter whose response function  $H(f)$  depends on the angle of signal arrival. Figure 1 is a schematic of a linear frequency-scanned array having  $N$  equally spaced elements, each identically excited by the jamming signal with spectrum  $S_1(f)$ . (The signal is shown here as a plane wave-front parallel to the array face.) The signal received by each element is fed into a tapped delay line, or a sinuous feed, which has identical time delays  $\tau$  between adjacent elements.

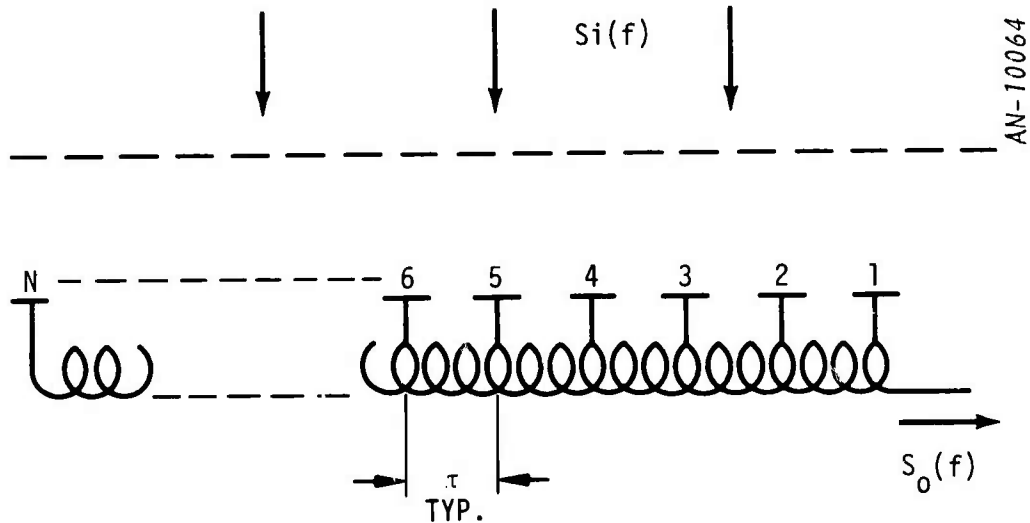


Figure 1. Schematic of a Linear Frequency-Scanned Array

The resulting output spectrum  $S_o(f)$  is given by

$$S_o(f) = AS_1(f) \sum_{n=0}^{n=N-1} e^{j2\pi n f \tau} = AS_1(f) e^{j\pi(N-1)f\tau} \left[ \frac{\sin N\pi f \tau}{\sin \pi f \tau} \right]$$

where A is a complex constant that relates the response of an element to the incident signal, and the exponential term represents the time delay from the array center. When these two factors (which can be eliminated by fixed adjustments) are disregarded, the frequency response of the array<sup>\*</sup> is given by

$$H(f) = \frac{S_o(f)}{S_i(f)} = \frac{\sin N\pi f\tau}{\sin \pi f\tau}$$

This function is plotted in Fig. 2. The main peaks occur at frequencies  $\hat{f} = m/\tau$ , where m is an integer. The peaks have spectral widths of  $1/N\tau$  (measured at  $2/\pi$  of the peak amplitude, or about 4 dB down).

A signal arriving from a direction other than normal to the array will produce a slightly different interelement delay (Fig. 3). The total time delay between adjacent elements  $\tau$  is given by

$$\tau = \tau_o + \frac{d}{c} = \tau_o + \frac{D}{c} \sin \theta$$

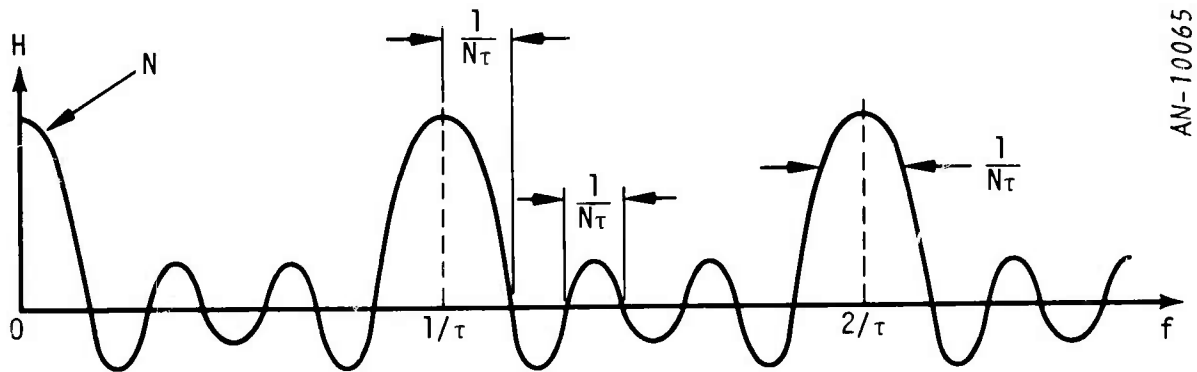
where  $\tau_o$  = delay-line time delay between adjacent elements  
 $D$  = element spacing  
 $d$  = signal path length difference between elements  
 $\theta$  = angle off boresight (normal to array face)  
 $c$  = propagation velocity

The frequency response function is accordingly expanded or contracted along the frequency axis.

For practical radar design, the value of  $\tau$  must be small enough that the main lobe spectral width  $1/N\tau$  is greater than the instantaneous

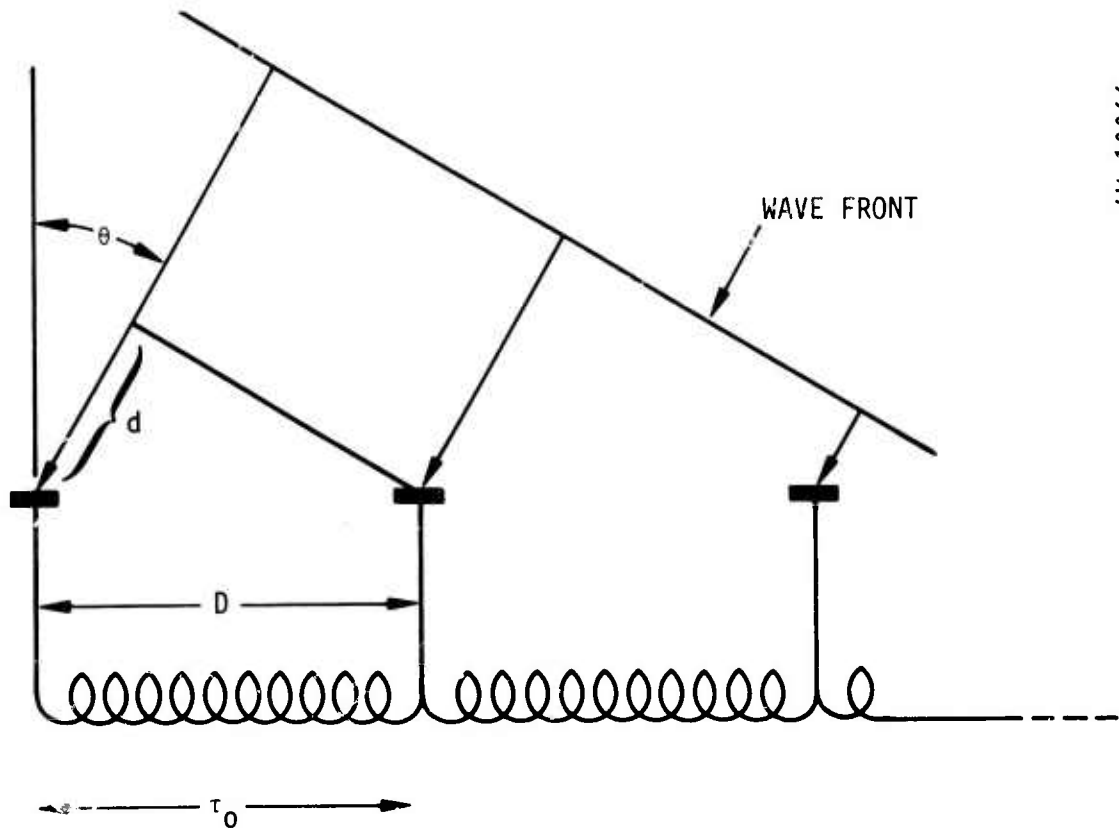
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\* The bandwidth limitations of individual array components, neglected here, will be considered later.



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Figure 2. Frequency-Response Function of a Frequency-Scanned Array  
(Drawn for  $N = 3$ )



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Figure 3. Effect of Angle of Arrival on Interelement Time Delay

signal bandwidth. This usually results in main lobe spacing  $1/\tau$  so large that only one lobe is contained within the usable bandwidth of the radar components.\* For example, with  $N = 100$  and a 10 MHz signal bandwidth, the maximum value of  $\tau$  is  $10^{-9}$  seconds and the main lobes must be spaced by at least 1000 MHz. With a 10-percent fractional bandwidth limitation (common for microwave components), using more than one main lobe is possible only at X band, and above. Consequently, this analysis considers principally the "single-lobe" case.\*\*

Consider a radar band in the vicinity of frequency  $f_o$ ; the broadside array beamwidth is approximately given by  $c/f_o ND$ . When the angle of signal arrival  $\theta$  changes by this amount, the frequency  $\hat{f}$  of a spectral peak in this band (corresponding to a particular value of the integer  $m$ ) will shift by approximately  $1/N_o$ \*\*\*. Thus, the antenna beamwidth is an angle that corresponds approximately to the width of the main spectral lobe of the array frequency response.

---

\* The conventional wrap-up factor  $W$  is given in terms of  $\tau$  by  $W = \tau(c/D)$ , where  $D$  is the element spacing and  $c$  is the propagation velocity. In terms of the nominal frequency  $f_o$  and wavelength  $\lambda_o$ ,  $W = \tau f_o (\lambda_o/D)$ .

\*\* In those cases where multiple lobes are available, an angular measurement can be made from each. When the resulting errors can be assumed to be independent, the overall error is reduced accordingly.

\*\*\* This is shown as follows: the shift  $\delta\hat{f}$  in the frequency peak due to a change in arrival angle from  $\theta_1$  to  $\theta_2$  is

$$\delta\hat{f} = \frac{m}{\tau_o + D/c \sin \theta_1} - \frac{m}{\tau_o + D/c \sin \theta_2} = \frac{mD/c (\sin \theta_2 - \sin \theta_1)}{(\tau_o + D/c \sin \theta_1)(\tau_o + D/c \sin \theta_2)}$$

Near boresight,  $\theta$  is small. Using the approximations  $\sin \theta \approx \theta$  and  $\tau_o \gg D/c \sin \theta$  gives

$$\delta\hat{f} = \frac{mD}{2c\tau_o} (\theta_2 - \theta_1)$$

$$\text{For } \theta_2 - \theta_1 = \frac{c}{f_o ND} = \frac{c\tau_o}{mND}$$

$$\text{then } \delta\hat{f} = \frac{1}{N\tau_o}$$

## B. SINGLE-BEAM MEASUREMENT

A conventional frequency-scanned array is configured to form a single beam as described above. The angle of signal arrival can be estimated by first finding the frequency of maximum spectral density output from the array  $f_m$  and then calculating  $\theta$  from

$$\sin \theta = \frac{c}{D} \left( \frac{m}{f_m} - \tau_o \right)$$

Figure 4a shows the array response function for an angle of arrival corresponding to the spectral peak  $\hat{f}$ . If the jamming spectral density is uniform over the frequency range illustrated ( $|S_1(f)|$  is constant), then the spectral density of the array output is as shown in Fig. 4b. In this case,  $f_m$ , the peak of  $|S_o(f)|$ , is equal to  $\hat{f}$ , the peak of  $H(f)$ , and the angle of arrival  $\theta$  can be accurately determined.

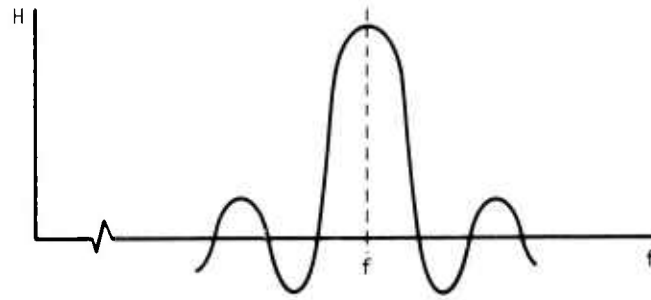
If the jamming spectrum is not uniform, however,\* the spectral density at the array output  $|S_o(f)|$  will not be the same as the magnitude of the array response function  $|H(f)|$ . In general,  $f_m$  will not equal  $\hat{f}$ , so that an angle-measurement error results. This is illustrated in Fig. 4d, which shows the spectral density of the array output for the jamming spectrum shown in Fig. 4c.

Error in measurements of angle of arrival, caused by spectral variation of the jammer, can easily amount to a significant fraction of the radar beamwidth. In the special case of a single-frequency CW jammer (or one with a very narrow bandwidth), angle indication would correspond

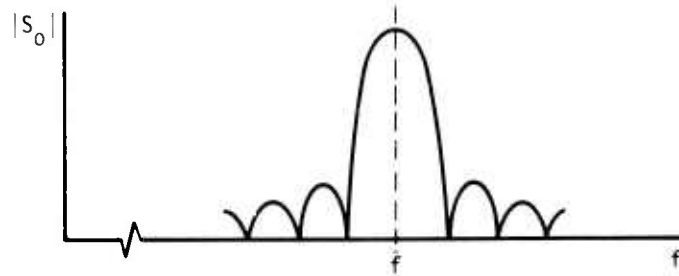
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\* The jamming spectrum  $S_1(f)$  is the Fourier transform of the segment of the jammer signal used for the measurement. The jamming power spectral density is given by  $\overline{|S_1(f)|^2}$ . Clearly, even when the power spectral density is uniform, the jamming spectrum can be nonuniform in individual observations.

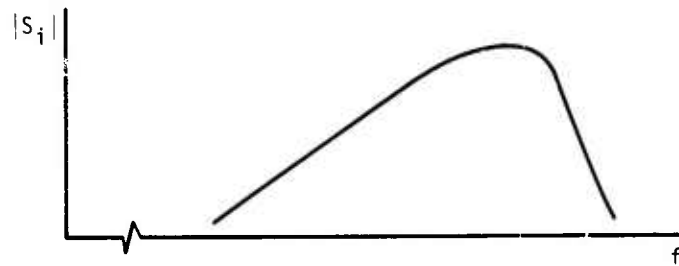
(a)



(b)



(c)



(d)

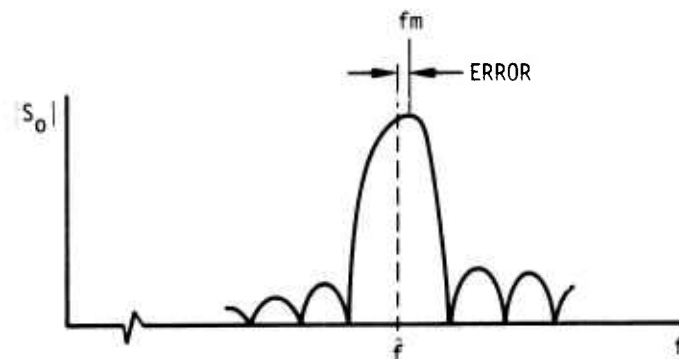


Figure 4. Spectra Illustrating Single-Beam Angle Measurement: (a) Array Frequency Response; (b) Output for Uniform Jamming Spectrum; (c) Sample Nonuniform Jamming Spectrum; (d) Output for Sample Jamming Spectrum

to jammer frequency, regardless of jammer location. (Such a jammer would be a main-lobe jammer, of course, only when at the angle corresponding to the jammer frequency.<sup>\*</sup>)

In practical measuring circuits, jammer signals may cause an additional component of angle-measurement error through an inaccurate determination of  $f_m$ . For example, if  $f_m$  were assumed to be midway between two frequencies (separated, for instance, by  $1/N\tau_0$ ) where the spectral densities were equal, asymmetry in the output spectral response peak resulting from a nonuniform jammer spectrum would cause a measurement error. Then, if the measurements that determine  $f_m$  were not made simultaneously, the variation of jammer power with time could cause additional error.<sup>\*\*</sup>

#### C. LOBE COMPARISON

The angle-measurement error caused by variations in jammer spectral density can be eliminated conceptually, and greatly reduced in practice, by using two arrays having slightly different frequency responses--the equivalent of forming two lobes at slightly different angles. This is illustrated by Fig. 5a, which shows the array frequency responses for two slightly different values of  $\tau$  when the same (or identical) array elements are used. The response peaks are separated by a frequency approximately equal to the spectral width of a peak, and the responses cross over (that is, they are equal) at about the 3 dB point of each peak.

A nonuniform jammer spectrum (such as shown in Fig. 5b) will cause the array output spectra to differ from the array frequency responses

---

<sup>\*</sup> ECM tactics designed especially to confuse frequency-scanned radars (e.g., multiple CW jammers, complex waveforms) are not considered in this report.

<sup>\*\*</sup> When measuring the angular position of non-jamming targets, fluctuations of target cross section can cause an error if the observations are not made simultaneously. Errors due to spectral variations of the signal return can be eliminated when the shape of this spectrum is known. The received spectrum can be assumed to be the same as the transmitted spectrum within the accuracy required here, at least for targets smaller than the length of the array.



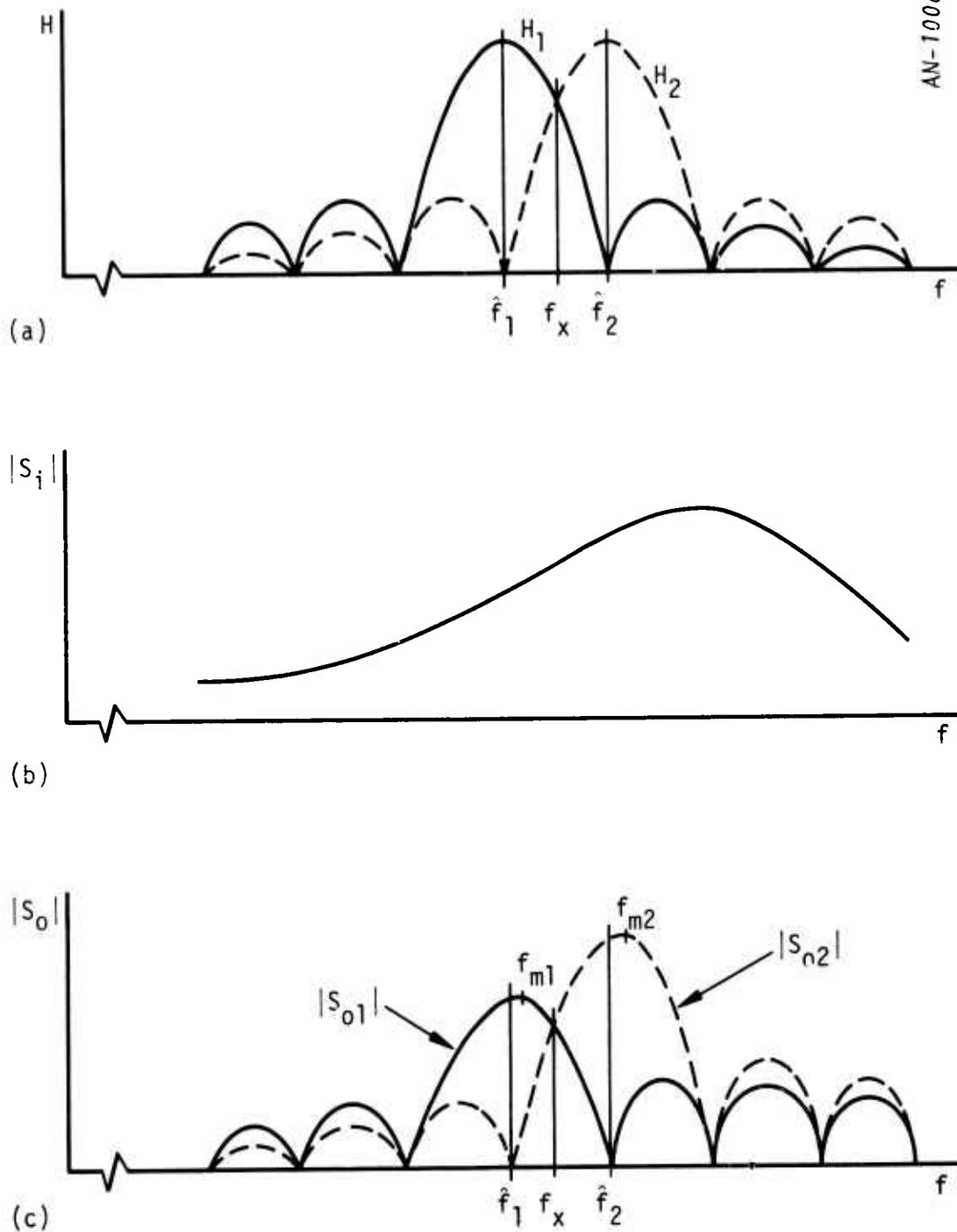


Figure 5. Spectra Illustrating Angle Measurement by Lobe Comparison:  
 (a) Array Frequency Responses; (b) Sample Nonuniform Jamming Spectrum; (c) Array Outputs for Sample Jamming Spectrum

and, in general, from each other (Fig. 5c). At the crossover frequency  $f_x$ , however, at which the two array responses are equal, the output spectral densities are always equal. Since  $f_x$  is a known function of the angle of signal arrival, measurement of this angle is essentially independent of the jammer spectrum.

Three techniques for obtaining the pair of array frequency responses described above are illustrated in Fig. 6. Figure 6a shows two identical arrays, positioned with their boresights differing by an angle approximately equal to the array beamwidth. With this configuration, the ratio of the frequency separation of the response peaks to the spectral width of the peaks varies approximately as  $\cos \theta$ .<sup>\*</sup> (This corresponds to the usual beam broadening for off-boresight scan angles, compared with the fixed angular separation of the beams.) The resulting variation in the crossover level with scan angle, together with the inconvenient mechanical layout, tend to make this configuration unattractive.

In Fig. 6b, the signal from each array element is divided between two sinuous feeds which have slightly different interelement delays. (A separate array of elements for each sinuous feed could be used also.) With this configuration, the width of the spectral peak increases approximately with the separation of the peak frequencies as  $\theta$  decreases. For a nominal crossover at  $2/\pi$  of peak amplitude, the proportionality varies with the factor

$$1 + \frac{2D/c \sin \theta}{\tau_{o1} + \tau_{o2}}$$

which would normally vary with  $\theta$  by less than  $\pm 5$  percent.

The ratio of the separation of the peak frequencies to the spectral peak width is independent of scan angle for the configuration shown in

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<sup>\*</sup> This result and others given in this section are derived in the appendix.

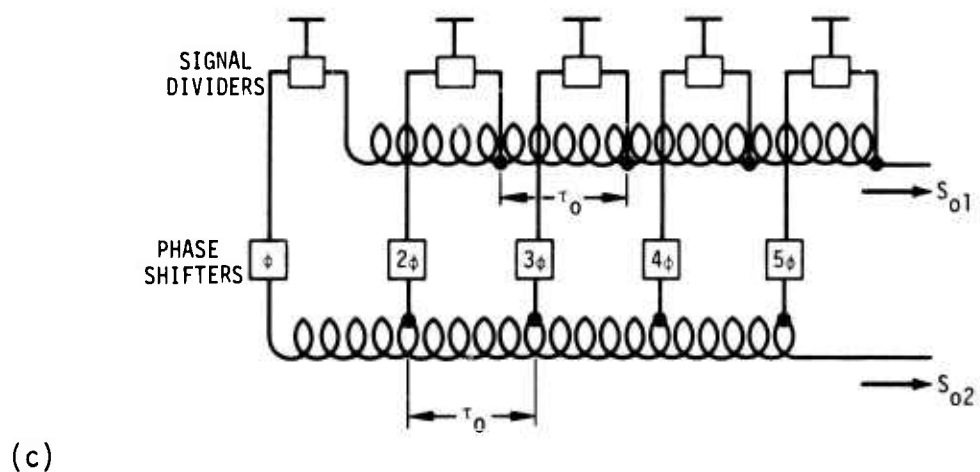
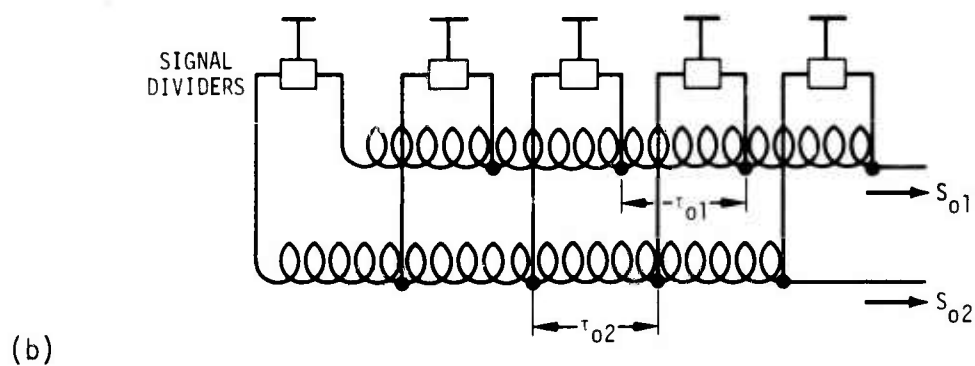
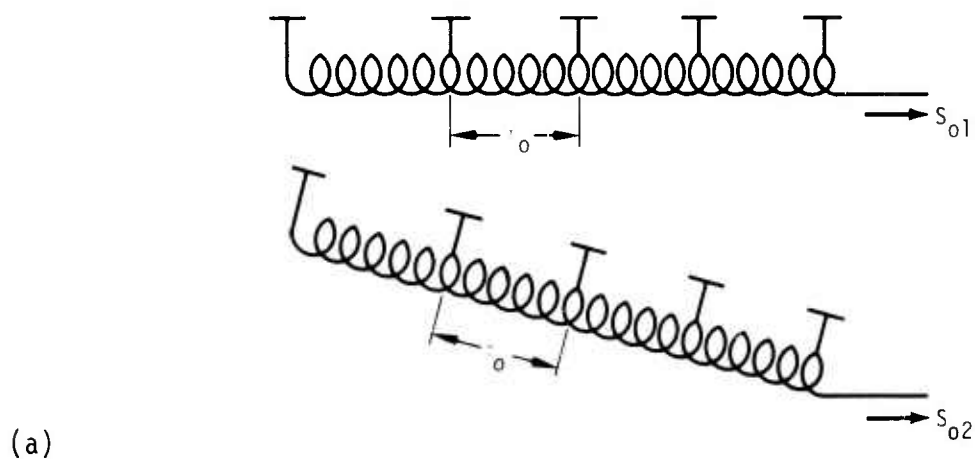


Figure 6. Techniques for Obtaining Signals for Lobe Comparison

Fig. 6c. Identical sinuous feeds are used, so that differential phase shifts must be applied to the signals fed to each from an element. The size of the differential phase shift for an element (shown here inserted at the input to one of the sinuous feeds) is proportional to the distance of the element from one end of the array. Choosing between this technique and the one described in the preceding paragraph would probably depend on details of the mechanical and electrical implementation.

Note that with each of these configurations only half the total power received by all the array elements is available in each array output. This contrasts with a conventional monopulse antenna which can give peak gain corresponding to the full aperture for each beam (provided that the beam crossover is at least 3 dB down). The full signal from both output channels is not available simultaneously with the monopulse antenna, as it is with the frequency-scanned array--hence the difference.

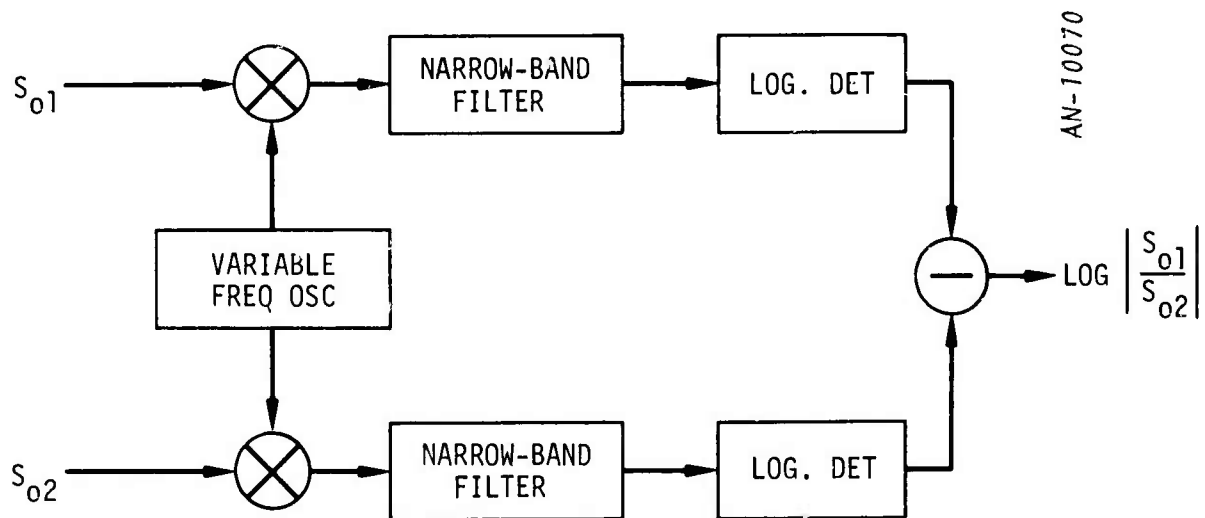
#### D. SIGNAL PROCESSING

In measuring jammer angular position with frequency-scanned arrays, the best receivers for lobe comparison resemble monopulse measuring systems.<sup>2</sup> The principal differences are that range gating is not needed and that narrow-band filters are used to select a spectral band corresponding to a small angular sector. The amplitude monopulse techniques described below apply here because a frequency-scanned array is analogous to an amplitude angle sensor. (Phase monopulse techniques could be used if the error signals were appropriately converted.<sup>2</sup>)

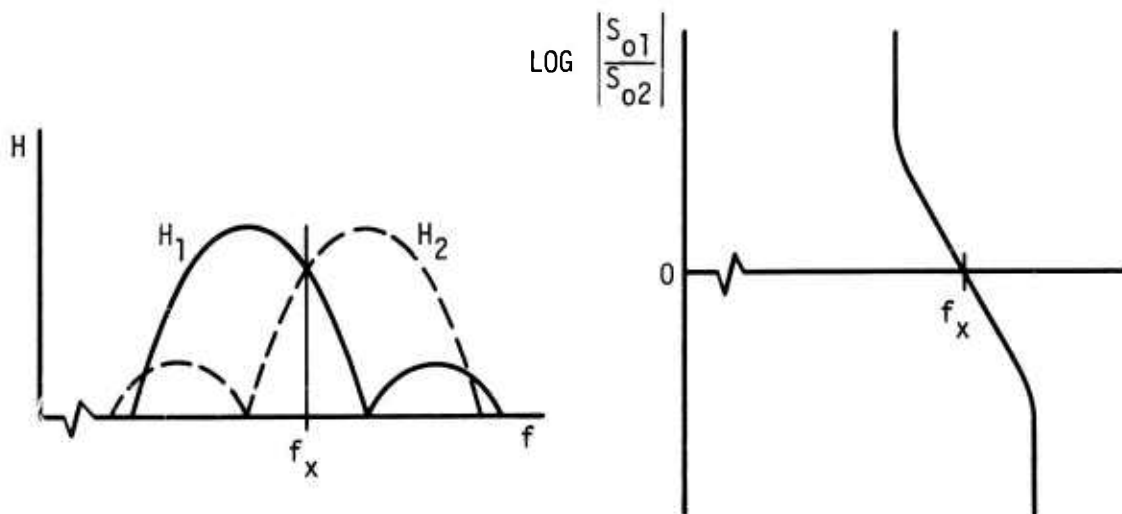
A receiver circuit for processing signals from a frequency-scanned lobe comparison array is shown in Fig. 7a. A pair of identical narrow-band\* filters is, in effect, tuned over the received spectrum by the variable-frequency oscillator that feeds the mixers. The logarithmic detectors reduce the dynamic range of the signals and also permit their ratio to be formed by a simple subtracting circuit.

---

\*The effect of finite bandwidth is discussed in Sec. II E.



(a)



(b)

(c)

Figure 7. Lobe-Comparison Receiver Using Amplitude Angle Detection:  
 (a) Block Diagram; (b) Array Response Functions; (c) Output Characteristic

Figure 7c shows the output signal amplitude as a function of the frequency to which the filter is tuned. Fluctuations due to jammer spectral variations are eliminated when the signal ratio is formed. With a suitably calibrated error curve (such as Fig. 7c), the angle of signal arrival can be found from a measurement at a frequency other than the crossover frequency  $f_x$ .

Another receiver employs sum and difference signals formed at RF and filtered as previously described (Fig. 8a). The difference/sum ratio is formed in the difference channel gain control amplifier by the action of the instantaneous automatic gain control (IAGC) signal from the sum channel. The normalized sum channel signal serves as a phase reference for the phase detector. This type of receiver is often preferred to the type shown in Fig. 7, because stability of the crossover frequency does not depend on accurate matching of the IF amplifiers and detectors.<sup>2</sup>

#### E. BANDWIDTH AND NOISE CONSIDERATIONS

In the preceding discussion of lobe-comparison receivers, the narrow-band filters were assumed to pass a very small portion of the signal spectrum--essentially a single frequency. This is analogous to pointing a mechanical antenna in a single direction (at a time).<sup>\*</sup> Since a wider bandwidth may be necessary or desirable for other reasons, its effect on angle measurement should be considered.

With reference to Fig. 7a, signal power out of the narrow-band filter in the top channel is given by

$$\int_{-\infty}^{\infty} |S_1(f)H_1(f)N(f)|^2 df$$

---

\* When a jammer is moved in angle, the power output of a frequency-scanned array, observed in a narrow frequency band, varies to form an antenna pattern in the same way as the output power from a stationary mechanical antenna. Widening the observation bandwidth in the frequency-scanned array will have the effect of widening the lobes of the antenna pattern.

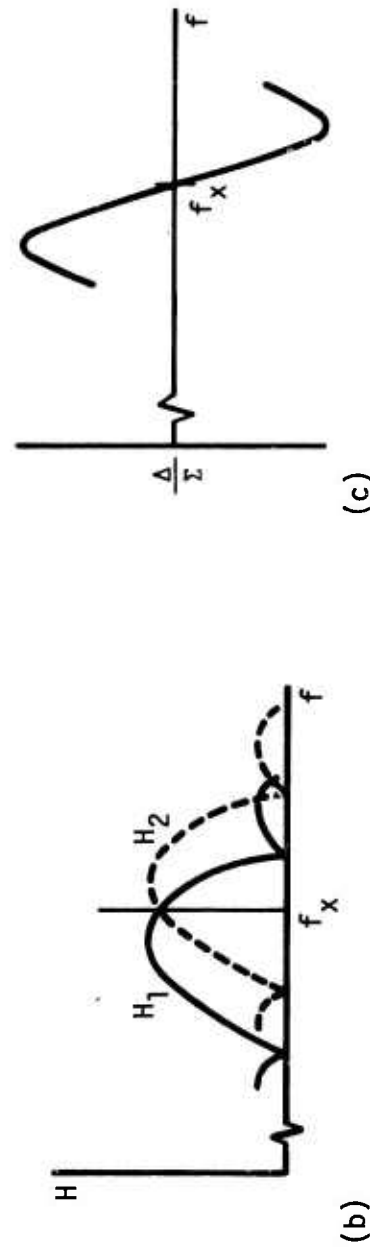
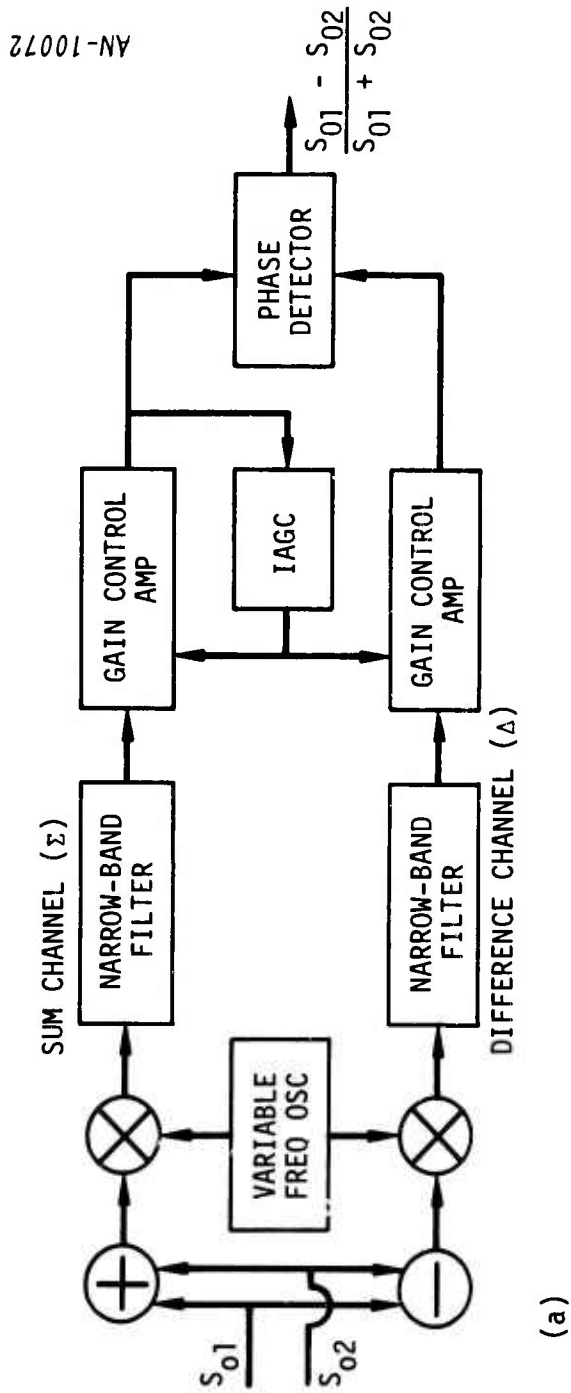


Figure 8. Lobe Comparison Receiver Using Sum Difference Angle Detection:  
 (a) Block Diagram; (b) Array Response Functions; (c) Output Characteristics

where  $H_1(f)$  is the array response function and  $N(f)$  is the response of the narrow-band filter. Assuming an "ideal" filter where  $N = 1$  for  $f_1 \leq f \leq f_2$ , and  $N = 0$  elsewhere, and normalizing with respect to the jamming power,

$$\hat{P}_{o1} = \frac{\int_{f_1}^{f_2} |S_1(f)|^2 H_1^2(f) df}{\int_{f_1}^{f_2} |S_1(f)|^2 df}$$

where  $\hat{P}_{o1}$  is the normalized output power.\*

It can be seen that  $\hat{P}_{o1}$  is simply a weighted average of  $H_1^2$ , and its value will therefore correspond to some value of  $H_1^2$  in the frequency range between  $f_1$  and  $f_2$ . When  $H_1^2$  is a monotonic function of  $f$  over this frequency range (Fig. 9), then  $\hat{P}_{o1}$  lies between  $H_1^2(f_1)$  and  $H_1^2(f_2)$ . Under the same condition,  $\hat{P}_{o2}$  lies between  $H_2^2(f_1)$  and  $H_2^2(f_2)$ .

A logarithmic function of the ratio  $\hat{P}_{o1}/\hat{P}_{o2}$  is formed by subtracting the log-detector outputs. (See Fig. 7a.) When the array response functions have opposite slopes, as they do between the frequency peaks (and as shown in Fig. 9), then  $\hat{P}_{o1}/\hat{P}_{o2}$  lies between  $H_1^2(f_1)/H_2^2(f_1)$  and  $H_1^2(f_2)/H_2^2(f_2)$ . Thus, the lobe comparison receiver will indicate a frequency (corresponding to an angle of arrival) within the range of the filter passband.\*\*

It is reasonable to calibrate the receiver by assuming a uniform jamming spectrum.\*\*\* The resultant indicated frequency  $f_c$  will be near

---

\* Note that  $H(f)$  is real.

\*\* A similar argument can be made for the sum-difference receiver shown in Fig. 8.

\*\*\* When tracking non-jamming target returns, the signal spectrum, which is known, can be used for calibration.



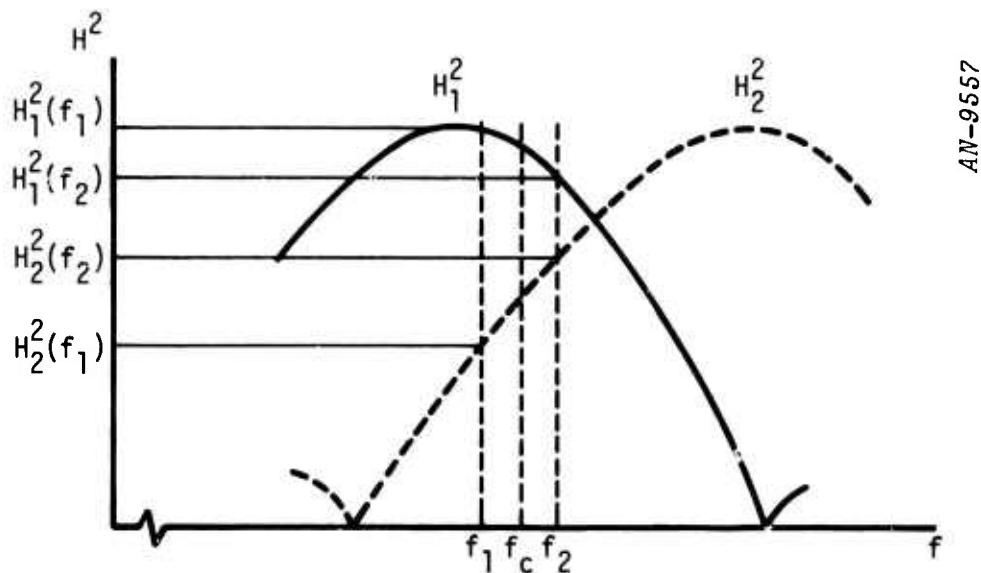


Figure 9. Effect of Finite Filter Bandwidth on a Lobe Comparison Receiver

the center of the filter passband. Hence, jammer spectral variations can cause a measurement error equal, at most, to the angle corresponding to one-half the bandwidth of the narrow-band filter. With the approximations used in Sec. II A, the angle-measurement error  $\delta\theta$  is related to the radar beamwidth  $\Psi$  by

$$\delta\theta = \frac{\Psi}{2} \left( \frac{\text{filter bandwidth}}{\text{width of spectral peak}} \right)$$

Angular accuracy of arrays is usually limited by mechanical and electrical tolerances to about one-thirtieth of the beamwidth. With the frequency-scanned lobe comparison configurations described, the error caused by jammer spectral variation can be kept below this limit if the filter bandwidth is less than one-fifteenth of the array spectral width. Since the signal bandwidth is normally less than a third of the array spectral width, the bandwidth need be narrowed by a factor of only about five when tracking jammers to obtain this accuracy.

A realistic filter has nonzero response outside the passband; consequently, severe variations in the jamming spectrum can conceivably cause a measurement error larger than half the filter bandwidth. For an error this large to occur, however, the jammer spectral density over the array response lobe width must vary by an amount greater than the skirt selectivity of the filter, possibly several tens of decibels, which is unlikely in a noise jammer. Still, the possibility of a specially designed jammer, perhaps with a comb spectrum, cannot be completely eliminated.

Since thermal noise power and jammer power are similarly affected by the narrow-band filter (assuming a uniform jamming spectrum), the jammer-to-noise ratio at the filter output is independent of bandwidth.\* The noise performance of the lobe-comparison receiver (See Figs. 7 and 8) is therefore independent of the filter bandwidth. The noise level can be further reduced by smoothing in the detector output circuit (not shown in the figure).

When a narrow filter bandwidth is used, the remaining portions of the spectrum between the spectral peaks can be used also. By using more than one filter circuit, such as those described, independent measurements of angular position can be made. The maximum number of such measurements is inversely proportional to the filter bandwidth, so that the variance (due to thermal noise) of the combined estimate of angular position varies as the square root of filter bandwidth. Since the jammer-to-noise ratio is high in cases of practical interest, a single filter circuit is usually adequate.

Note, however, that this technique is analogous to the optimum method of using conventional antennas to estimate the angular position

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\* For non-jamming targets,  $S/N$  varies approximately as bandwidth for bandwidths less than that of a matched filter, because signal power increases approximately with bandwidth squared in this range.

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of noise sources in the presence of thermal noise.<sup>3</sup> The principal difference is that the usable bandwidth is limited to about  $1/N\tau_0$  by the frequency response of the frequency-scanned array.

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### III. EXAMPLE: PHASE-FREQUENCY ARRAY RADAR

The radar chosen to illustrate how the lobe-comparison technique can be incorporated into multifunction radars that must be capable of surveillance, normal target tracking, and jammer-angle measurement, is a planar array radar, which uses frequency scanning in azimuth and phase scanning in elevation. The configuration shown in Fig. 10 is presented as an example and not as practical radar design.

The array consists of 36 radiating elements, fed in rows of 6 each by six sinuous feeds to provide the frequency scanning in azimuth. The feeds in the upper three rows produce a slightly different array frequency response from that produced in the bottom three rows, a result of either different interelement time delays or different phase shifts (described in Sec. II C).

Beam steering in elevation is produced by electronically adjusting the phase shifters at the end of each row to give the required vertical-phase taper in the array. Following each phase shifter is a duplexer (TR) with which the array can be connected to the transmitter power amplifier or connected through preamplifiers to the receiver beam-forming circuits.

The phase-shifted signals from the sinuous feeds combine in the receiver to form a sum, or reference, beam and azimuth- and elevation-difference beams. The sum beam is formed by an in-phase addition of the six signals; whereas, the elevation-difference beam is formed by applying a phase taper (as in a conventional monopulse receiver). Having the two response functions broadens the sum beam somewhat in azimuth and correspondingly reduces the peak gain.\* (The same effect is found in dish reflectors with 4-horn monopulse feeds.)

To form the azimuth-difference beam, the upper and lower groups of rows are first summed individually, thereby combining signals corresponding

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\*The transmit beam, also, is broadened in azimuth and reduced in peak gain.

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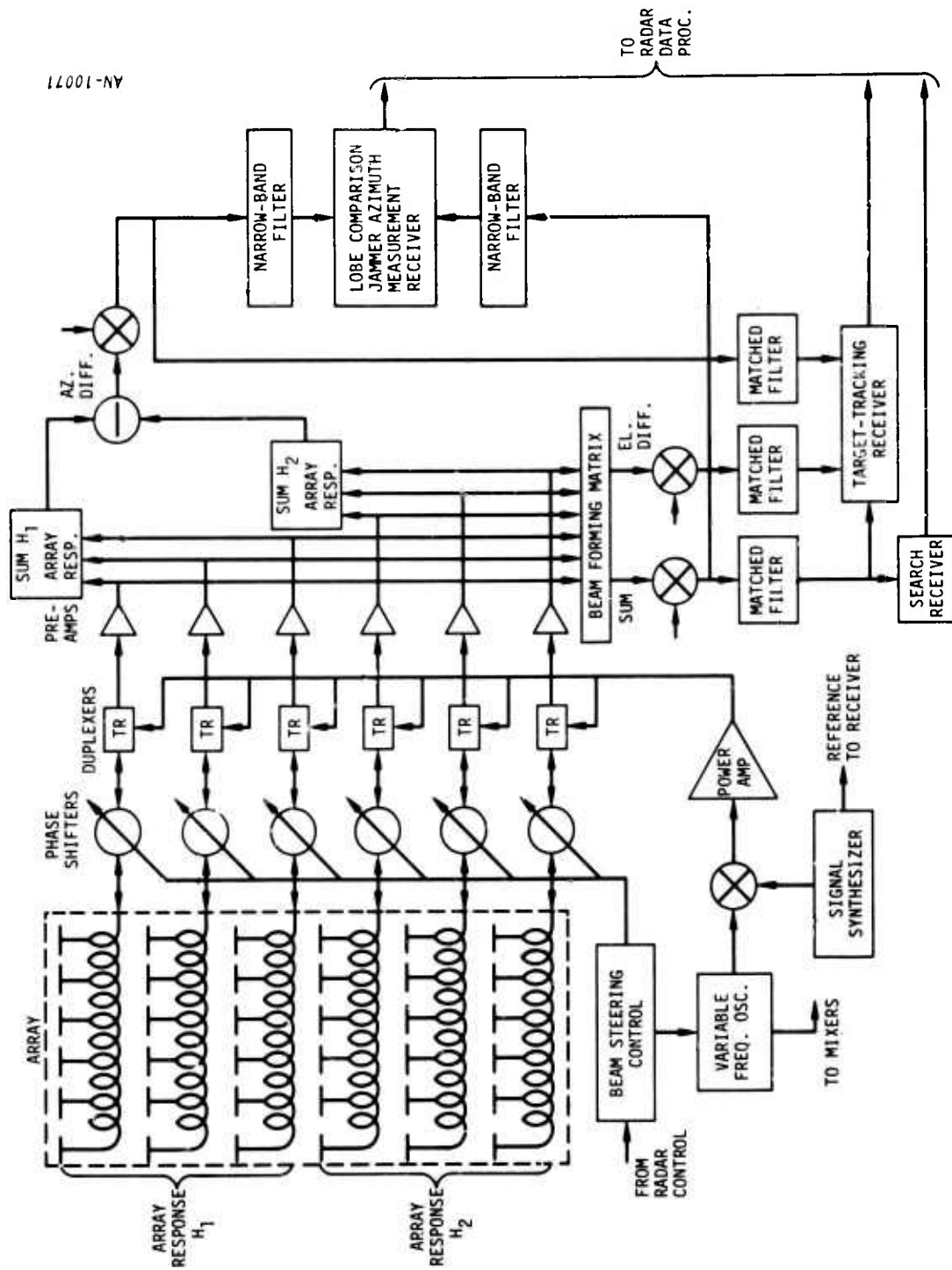


Figure 10. Phase-Frequency Array Radar With Lobe Comparison for Measuring Jammer Azimuth

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to each array response function. The resulting sum signals are then subtracted to form a difference signal comparable to that in the lower channel in Fig. 8a.

Elevation sidelobes can be reduced by amplitude-weighting the low signals. Identical weighting for every beam can be applied by adjusting the preamps, or appropriate weighting for each beam type can be applied individually in the beam-forming circuits. Azimuth sidelobes are reduced by identically tapering the excitation of the elements in each row--for example, by adjusting the coupling between each element and the sinuous feed.

The three outputs of the beam-forming circuits are mixed with a signal whose frequency is adjusted to produce the desired azimuth-steering of the beams. (The same variable-frequency signal generates the transmitted signal.) The resulting IF signals pass through filters, matched to the transmitted waveform, and into conventional search and monopulse-tracking receivers.\* The elevation angle of jammers is also measured by the tracking receiver.

The sum and azimuth-difference signals pass through narrow-band filters also and into a lobe-comparison receiver for measuring the azimuth of jammers. The over-all circuit configuration for this function is the same as shown earlier in Fig. 8a.

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\* Note that the lobe comparison receiver in Fig. 8a is similar to a monopulse tracking receiver except for the narrow-band filter and the absence of range gating. As stated previously, the monopulse receiver can accurately measure angle when the signal spectrum is known.

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IV. CONCLUSIONS

When measuring the angular positions of jammers, conventional frequency-scanned radars can produce errors as large as half the beam-width because of variation of jammer power with time or frequency; the lobe comparison technique can reduce these errors to below the accuracy limit set by mechanical and electrical tolerances in the array. Compared with a conventional monopulse array radar, the application of lobe comparison to a phase-frequency scanned array radar requires only a small amount of additional equipment. Thus, in the angular tracking of jammers, frequency-scanned radars can compete with the (usually) more complex and costly phase-scanned radars.

## APPENDIX

### SPECTRAL RELATIONSHIPS IN LOBE-COMPARISON FREQUENCY-SCANNED ARRAYS

In a frequency-scanned array using lobe comparison to measure jammer angular position, the null angle corresponds to the crossover frequency  $f_x$  of two array frequency response functions. As the angle of signal arrival  $\theta$  changes, the frequencies of the two response peaks  $\hat{f}_1$  and  $\hat{f}_2$  (see Fig. 5a) move in the same direction. The crossover frequency  $f_x$ , which lies between  $\hat{f}_1$  and  $\hat{f}_2$ , varies smoothly with  $\theta$ ; depending on the array configuration, however, the level of the crossover (relative to the peak response level) may vary with  $\theta$ , possibly causing a change in the measurement sensitivity.

Crossover level is a function of the ratio of the separation of the frequency peaks  $\Delta\hat{f} = \hat{f}_2 - \hat{f}_1$  to the sum of the half-bandwidths of the spectral lobes  $BW = 1/2 (BW_1 + BW_2) = 1/2N (1/\tau_1 + 1/\tau_2)$ . The ratio of these quantities  $\Delta\hat{f}/BW$  is calculated below, as a function of  $\theta$ , for the three array configurations described in Sec. II C and illustrated in Fig. 6.

#### A. IDENTICAL ARRAYS WITH OFFSET BORESIGHTS (See Fig. 6a)

Let the boresight of the two arrays be offset by angles of plus and minus  $\delta$  from  $\theta = 0$ :

$$\begin{aligned} \frac{\Delta\hat{f}}{BW} &= \frac{\frac{m}{\tau_o + D/c \sin(\theta - \delta)} - \frac{m}{\tau_o + D/c \sin(\theta + \delta)}}{\frac{1}{2N[\tau_o + D/c \sin(\theta - \delta)]} + \frac{1}{2N[\tau_o + D/c \sin(\theta + \delta)]}} \\ &= \frac{2Nm[D/c \sin(\theta + \delta) - D/c \sin(\theta - \delta)]}{2\tau_o + [D/c \sin(\theta + \delta) + D/c \sin(\theta - \delta)]} \end{aligned}$$



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Since  $\delta$  is small,  $\sin(\theta \pm \delta) \approx \sin \theta \pm \delta \cos \theta$ , giving

$$\frac{\Delta f}{BW} \approx \frac{2NmD\delta}{c} \times \frac{\cos \theta}{\tau_o + D/c \sin \theta} \quad (\delta \ll 1)$$

In practical arrays  $\tau_o$  is of the order of  $10 D/c$ , resulting in the principal angular dependence of  $\Delta f/BW$  being as  $\cos \theta$ . If the angle between the array boresights  $2\delta$  equals the broadside beamwidth  $\lambda_o/ND = c\tau_o/NmD$  (see Sec. II A), then,

$$\frac{\Delta f}{BW} \approx \cos \theta \quad \left\{ \begin{array}{l} \delta \ll 1 \\ D/c \ll \tau_o \\ 2\delta = \frac{c\tau_o}{NmD} \end{array} \right.$$

## B. ARRAYS WITH DIFFERENT INTERELEMENT DELAYS (See Fig. 6b)

Let the interelement time delays for the two arrays be  $\tau_{o1}$  and  $\tau_{o2}$ :

$$\begin{aligned} \frac{\Delta f}{BW} &= \frac{\frac{m}{\tau_{o2} + D/c \sin \theta} - \frac{m}{\tau_{o1} + D/c \sin \theta}}{\frac{1}{2N(\tau_{o2} + D/c \sin \theta)} + \frac{1}{2N(\tau_{o1} + D/c \sin \theta)}} \\ &= \frac{Nm(\tau_{o1} - \tau_{o2})}{1/2(\tau_{o1} + \tau_{o2}) + D/c \sin \theta} \end{aligned}$$

As stated above, the average interelement delay  $1/2(\tau_{o1} + \tau_{o2})$  is of the order of  $10^*$  times  $D/c$ . Thus,  $\Delta f/BW$  will vary less than  $\pm 5\%$  over the entire range of scan angles. To obtain a crossover at the half-bandwidth points of the response peaks at boresight (i.e.,  $\Delta f/BW = 1$ ) requires that

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\* Corresponds to the array wrap-up factor (see Sec. II A).

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$$\tau_{o1} - \tau_{o2} = \frac{\tau_{o1} + \tau_{o2}}{2Nm}$$

C. ARRAYS WITH DIFFERENT PHASE SHIFTS (see Fig. 6c)

Let incremental phase shifts of plus and minus  $\phi$  be applied to arrays 1 and 2, respectively:

$$\hat{f}_1 = \frac{m}{\tau_o + \frac{\phi}{2\pi\hat{f}_1} + \frac{D}{c} \sin \theta} = \frac{m - \frac{\phi}{2\pi}}{\tau_o + \frac{D}{c} \sin \theta}$$

and

$$BW_1 = \frac{1}{N(\tau_o + \frac{\phi}{2\pi\hat{f}_1} + \frac{D}{c} \sin \theta)} = \frac{m - \frac{\phi}{2\pi}}{mN(\tau_o + \frac{D}{c} \sin \theta)}$$

Expressions for  $\hat{f}_2$  and  $BW_2$  are similarly derived. The ration  $\Delta\hat{f}/BW$  is given by

$$\frac{\Delta\hat{f}}{BW} = \frac{\frac{m + \frac{\phi}{2\pi}}{\tau_o + \frac{D}{c} \sin \theta} - \frac{m - \frac{\phi}{2\pi}}{\tau_o + \frac{D}{c} \sin \theta}}{\frac{m + \frac{\phi}{2\pi}}{2mN(\tau_o + \frac{D}{c} \sin \theta)} + \frac{m - \frac{\phi}{2\pi}}{2mN(\tau_o + \frac{D}{c} \sin \theta)}} = \frac{N\phi}{\pi}$$

which is independent of  $\theta$ . To obtain a crossover at the half-bandwidth points, the interelement differential phase shift  $2\phi$  must equal  $2\pi/N$ . The total phase-shift difference across the array is  $N$  times this value, or  $2\pi$ .

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| <p>Frequency-scanned array radars may be preferred to phase-scanned radars because they tend to be cheaper and simpler. When conventional frequency-scanned radars are used to measure the angular position of jammers, however, large errors can result. These can be eliminated with the lobe-comparison technique.</p> <p>The response of a frequency-scanned array to a jammer can be compared with that of a filter whose characteristics depend on the angle of signal arrival. Variations of jammer power with frequency and time can distort the array output spectrum to cause measurement errors. With lobe comparison, the array is divided to produce two slightly different frequency response functions (equivalent to two beams, like those in a monopulse radar). The angle of signal arrival is estimated by comparing the response of the two arrays in a narrow frequency band. If the band is sufficiently narrow, measurement error can be reduced to a satisfactory level with no penalty in noise performance. In practice, a bandwidth about one-fifth that of the radar signal reduces error to less than the usual fixed-error limit of the array.</p> <p>Lobe comparison can be applied to frequency-scanned radars, including phase-frequency-scanned arrays, with only a small increase in cost and complexity. The improved performance in measuring jammer angular position compares well with that of phase-scanned arrays.</p> |   |   |

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| Angular Measurement Accuracy |        |    |        |    |        |    |
| Array Radars                 |        |    |        |    |        |    |
| Complexity                   |        |    |        |    |        |    |
| Cost                         |        |    |        |    |        |    |
| Frequency Response           |        |    |        |    |        |    |
| Frequency Scanning           |        |    |        |    |        |    |
| Lobe Comparison              |        |    |        |    |        |    |
| Monopulse                    |        |    |        |    |        |    |
| Noise Jammers                |        |    |        |    |        |    |